Schroeter's Rule and the Modification of Lunar Crater Impact Morphology

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The 1:1 correspondence of crater rim volume to true crater volume allegedly observed for many lunar craters has been explained previously by the theories of explosive meteoritic impact. In fact, the equality occurs too infrequently to be of genetic significance. The ratio characteristically is about 0.4 to 0.8 for small, freshly formed lunar and terrestrial craters, but it is well over 1.0 in older and larger lunar craters. Most of the moon's craters are of explosive impact origin and acquire a specific, predictable shape governed by the law of allometric growth. Fresh crater dimensions and volumes are related by expressions of the form $y = ax^b$. Up to a diameter of 10 to 20 km, craters are stable landforms. Larger craters deform isostatically and are further modified by extrusive activity. Application of an allometric fresh crater shape model to Tycho suggests that even the most youthful appearing large lunar craters do not long retain their initial rim and true crater volumes, but that they begin to deform soon after impact.

INTRODUCTION

From measurements of lunar crater surface dimensions, *Schroeter* [1791–1802] concluded that the volume of the crater rim usually equaled that of the true crater: $V_r = V_t$ (Figures 1 and 2, Table 1); definitions are those of *Baldwin* [1963]). Known as 'Schroeter's rule,' this observation generally is believed to reflect an explosive origin of the lunar craters. It is argued that any given true crater volume V_t excavated by explosive energy must be complemented by an equal volume V_r of displaced ejecta in the surrounding rim. Since most authorities view meteoritic infall as a more likely source of explosive energy than, for example, volcanic activity, Schroeter's observation has been interpreted further as reflecting an impact genesis for the lunar craters [*Baldwin*, 1963]. The purpose of this paper is to demonstrate that the original formulation of the 'rule' has little genetic application because (1) the initial V_r/V_t of impact craters is not 1.0, and (2) post-impact endogenic activity has altered the initial impact configuration of most lunar craters over 10–20 km in diameter.

INADEQUACIES OF SCHROETER'S RULE

The currently accepted interpretation of Schroeter's rule is deficient in two respects. First, the V_r/V_t proportion of any freshly formed explosive impact crater on the moon probably is much less than 1.0. Shallow-burst terrestrial explosion craters provide reasonable analogs with freshly formed meteorite craters [*Roberts*, 1966; *Short*, 1965]. The V_r/V_t ratios of terrestrial experimental craters are of the order of 0.4 to 0.8 rather than 1.0 [*Carlson and Roberts*, 1963; *Carlson and Jones*, 1965]. The observed excess of V_t over V_r, or V_e, is due to the displacement of much ejecta beyond the edge of the exterior crater rim slope and to vertical and lateral displacement of bedrock adjacent to the crater depression. During a lunar impact, ejection of material beyond the confines of the moon's gravity would further diminish the V_r/V_t proportion.

Second, Schroeter's original observation is applicable only to the large flat-floored craters to which his observations were restricted by the limitations of his equipment. Two recent lunar crater V_r and V_t estimates over a much wider D_r range reveal no concentration of V_r/V_t values at 1.0. *Baldwin* [1963] examined over 550 terrestrial explosion and meteorite craters and *fresh appearing* lunar craters. His V_r/V_t data vary systematically with D_r up to 10 km D_r, whereupon V_r/V_t increases at a significantly greater rate from 0.60 to 0.85 at a D_r value of 100 km. V_r/V_t does not approach 1.0 until D_r exceeds 200 km (Figure 3). According to these

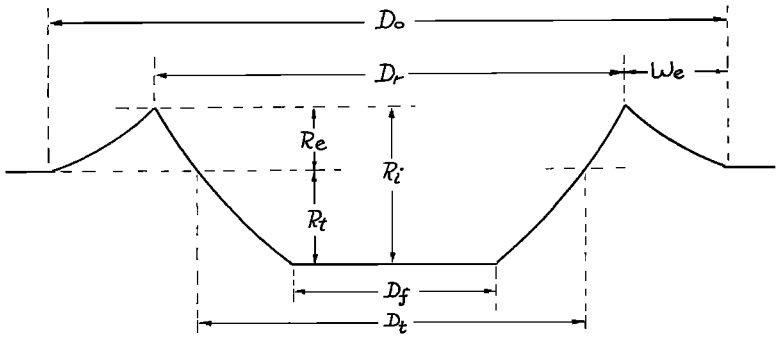

Fig. 1. Eight linear components of crater surface geometry.

data, the best lunar analogs of terrestrial experimental explosion craters are the youthful-appearing lunar craters up to about 10–20 km D_r. *Abrams* [1966] computed V_r and V_t for 35 lunar craters ranging in D_r from 15 to 110 km and *varying greatly in degree of apparent freshness*. Although a V_r/V_t value of 1.0 does bisect the V_r/V_t:D_r distribution, no significant clustering of V_r/V_t values occurs within the observed V_r/V_t range of 0.2 to 5.5. A restricted range of low (0.4 to 0.8) V_r/V_t values characterizes the younger, but not the older, appearing lunar craters. Many of the moon's craters that may have originated as meteorite impacts do not at present have the morphometry diagnostic of known explosive craters.

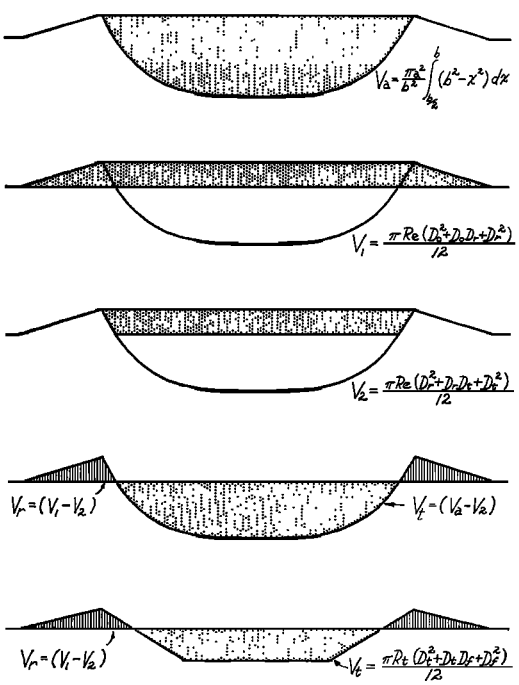

Fig. 2. Six crater volumes. Bottom profile is modified crater. Others are fresh impact craters. Terms defined in text or in Table 1. Diagrams not to scale.

Explosive Impact Craters and Allometric Growth

The volumetric data of Baldwin and Abrams suggest a critical variation of lunar crater shape with crater diameter and relative age. The relationship is examined further by an analysis of the linear crater dimensions R_i and R_e. R_i:D_r and R_e:D_r data for over 600 *fresh appearing* lunar and terrestrial craters were plotted on two logarithmic charts in the manner of *Baldwin* [1963, 1965]. Neither of the resulting point distributions is curvilinear, as Baldwin has suggested. Rather, each data plot is better fit by two linear expressions.

$$R_i = 0.155 D_r^{0.95} \quad (1)$$

and

$$R_e = 0.048 D_r^{0.95} \quad (2)$$

fit the respective data from a D_r of about 10 meters to a D_r of about 15 km (crater dimensions are in kilometers for all equations), whereupon *both* distributions change abruptly to trends with much lower slopes:

$$R_i = 0.880 D_r^{0.35} \quad (3)$$

$$R_e = 0.160 D_r^{0.53} \quad (4)$$

The better fit to the logarithmic R_i:D_r and R_e:D_r distributions of the smaller and fresh

TABLE 1. Eight Crater Surface Dimensions and Six Volumes
See Figures 1 and 2.

Surface Dimensions	Volumes
D_0, over-all diameter	V_a, apparent crater delimited by D_r, R_i (and D_f if a flat floor is present)
D_r, rim crest diameter	V_1, frustum delimited by D_0, D_r, and R_e
D_t, true crater diameter	V_2, frustum delimited by D_r, D_t, and R_e
D_f, flat interior floor diameter	V_t, true crater, below foot of exterior slope, or $(V_a - V_2)$. See also Figure 2.
R_i, interior relief ('crater depth')	V_r, rim, above foot of exterior slope, or $(V_1 - V_2)$
R_e, exterior relief ('rim height')	V_e, excess of true crater volume over rim volume or $(V_t - V_r)$
R_t, true crater interior relief	
W_e, exterior rim slope width	

appearing lunar craters afforded by linear equations rather than curvilinear expressions is a subtle but critical distinction. It suggests that the shapes of fresh impact craters form in accordance with a dimensional relationship known as the law of allometric growth. Originally formulated by biologists, the law states that the rate of relative growth of an organ (y) is a constant function of the rate of relative growth of the total organism (x) such that

$$y = ax^b \qquad (5)$$

where a and b are constants and a is always positive. The expression describes a linear regression for paired variables x and y on double logarithmic scales. The law of allometric growth can be reformulated to apply with equal validity to phenomena that 'grow' in a much shorter time than the more familiar biological allometric systems [*Nordbeck*, 1965]. In such cases, the individuals in the system must have a similar, specific shape, but they may vary in size.

The smaller lunar craters possess all three characteristics of allometric phenomena. First, *freshly formed* lunar *and* terrestrial craters appear to have similar shapes. The author has fit half of a hemi-ellipse both to a composite profile of the Arizona Meteorite Crater (less post-impact sediments) [*Batson*, 1964], and to the profile of a fresh 17 km D_r lunar crater [*Lunar Section Reports*, 1962–1964]. *Jeffreys* [1959] and *Warner* [1961*a*] have also observed that part of the hemi-ellipse best fits, respectively, the few comparatively uneroded terrestrial meteorite craters and the shape of many large, fresh lunar craters. The second allometric criterion, a linear log-log distribution of two descriptive variables, is fulfilled by equations 1 and 2. Finally, the slopes of the log-log distributions must closely approximate the ratio of the variables' dimensionalities. Since R_t, R_e, and D_r all are linear quantities possessing dimensionalities of 1 [*Bridgman*, 1931], the slopes of the R_t:D_r and R_e:D_r distributions should be 1.0. The measured slopes are 0.95, or close to the ideal ratio for craters under 10–20 km D_r.

Woldenberg [1966] points out that log-log regressions result from stochastic processes and hence can reflect conditions of maximum probability of energy expenditure, or the equilibrium of a physical configuration with its formative processes. It is known that the terrestrial craters described by equations 1 and 2 formed explosively. Consequently, terrestrial and small lunar craters whose R_t and R_e follow the linear, allometric equations 1 and 2 all were formed by

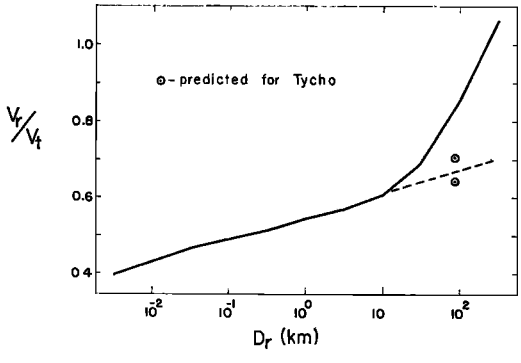

Fig. 3. Observed V_r/V_t:D_r distribution from data of *Baldwin* [1963] for fresh appearing terrestrial and lunar craters. Values for Tycho predicted by fresh explosion crater model derived in text.

a similar process. They are equilibrium landforms of explosion, and they reflect the most efficient expenditure of explosive energy possible under physical constraints imposed upon near-surface explosions. Furthermore, since terrestrial craters described by the allometric expressions (1) and (2) are known to be unmodified, then smaller lunar craters, under 10–20 km D_r, also must reflect their original explosive impact configurations.

Deformation of Impact Crater Shape

Allometric theory furnishes a useful criterion of impact crater freshness and suggests that most large lunar craters have lost their initial impact shape. Presumably, the intrinsic characteristics of the near-surface explosive impact process are invariable regardless of the energy level involved. The shape of the resulting craters will always be approximately the same equilibrium configuration. The R_i and R_e of freshly formed *or* unaltered explosive impact craters should follow only the allometric expressions (1) and (2) regardless of crater diameter.

The R_i and R_e values measured for most lunar craters over 10–20 km D_r *are much too low,* however, to fit (1) and (2). Morphologically, these larger craters resemble in many ways the smaller craters described by the allometric expressions. Presumably they also originated by explosive impact and initially possessed R_i and R_e values that followed (1) and (2). These allometric values have since diminished, resulting in the prominent 'elbows' in log-log distributions of V_r/V_i:D_r, R_i:D_r, and R_e:D_r at 10–20 km D_r (Figures 3 and 4). *Nordbeck* [1965] states

> . . . the size of a growing individual having a special shape always has an upper limit. If the individual exceeds this limit, its shape must be deformed. (For example) . . . a mountain much more than 10,000 meters in height must change shape due to structural failure of its base. . . . This deformation can easily be recognized in a diagram with logarithmic scales because there is a break in the point distribution corresponding to the change from one shape to another.

The data plots of surface dimensions and volumes describing lunar craters are classic examples of this particular type of shape change. A similar 10–20 km D_r threshold diameter range occurs in the R_i:D_r relationship for ten

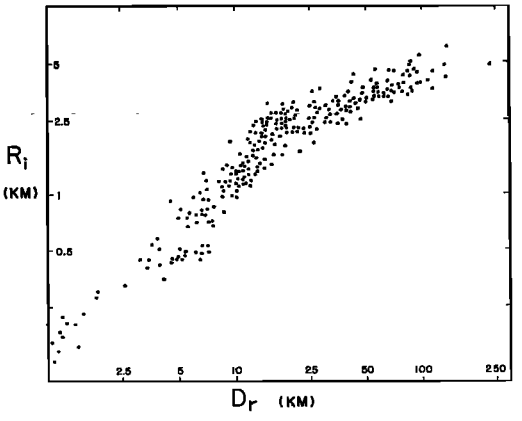

Fig. 4. Depth (R_i) to diameter (D_r) distribution of 250 fresh appearing lunar craters, showing approximate 15 km threshold D_r value separating modified from unaltered craters.

Canadian 'astroblemes.' This suggests further that large impact craters have undergone significant post-impact deformation [*Dence,* 1965].

Prediction of Fresh Impact Crater Volumes

It is desirable to predict the original equilibrium surface geometry of large lunar craters that at present exhibit only a modified configuration. If correctly estimated, resulting values of linear dimensions R_i, R_e, and D_0 will generate V_r, V_i, V_e, and V_r/V_i quantities characteristic of the explosive impact cratering process. Providing that shape deformation of the larger craters has not occurred *during* the impact event itself, allometric theory justifies a direct linear extrapolation of (1) and (2) to obtain the original R_i and R_e values.

D_0 (Figures 1 and 2) marks the topographically expressed outer limit of structurally uplifted bedrock comprising the crater rim [*Carlson and Jones,* 1965]. The linear log-log relationship between observed D_0 and D_r,

$$D_0 = 1.40 D_r \qquad (6)$$

has little scatter and is allometric throughout. There is no change in the slope of 1.0 at 10–20 km D_r. The horizontal crater dimensions are much less affected by post-impact disturbances than R_i and R_e are.

An unmodified impact crater does not have a flat floor. The half hemi-elliptical profile discussed above is adopted as the approximate

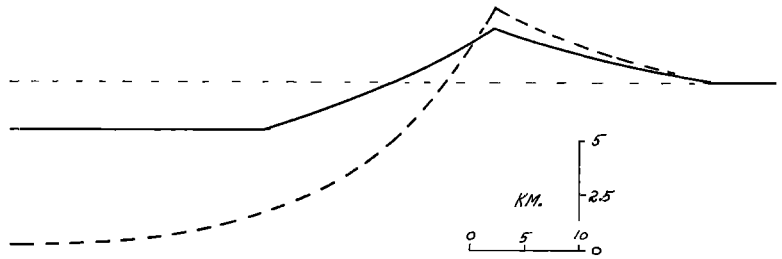

Fig. 5. Observed, modified (solid), and predicted, initial impact (dashed) profiles of the crater Tycho.

shape of the apparent impact crater depression (Figure 1, Table 1). It is fitted to the observed D_r and the R_i predicted from (1) by computing

$$a = \sqrt{\frac{(D_r/2)^2(2R_i)^2}{(2R_i)^2 - R_i}} \qquad (7)$$

by means of the general formula for the ellipse, where a is the semimajor axis of the fitted ellipse, and

$$b = 2R_i \qquad \text{the semiminor axis} \qquad (8)$$

True crater depth (R_t) and diameter (D_t) (Figure 1) are read from the profile. From (7) and (8), the volume of the initial apparent crater, V_a (Table 2, Figure 2), is computed by evaluating

$$V_a = \frac{\pi a^2}{b^2} \int_{b/2}^{b} (b^2 - x^2)\, dx \qquad (9)$$

Then, from Figure 2 and Table 1,

$$V_r = (V_1 - V_2) \qquad (10)$$

and

$$V_t = (V_a - V_2) \qquad (11)$$

Results for Tycho

The existing morphometry of the fresh appearing lunar crater Tycho contrasts markedly with that predicted by the fresh impact crater model derived above (Figure 5, Tables 2 and 3). From data of *MacDonald* [1931], *Whitaker et al.* [1963], *Kopal et al.* [1965], and *Kopal* [1966], the author has estimated Tycho's present V_r/V_t. The result, 1.99 (Table 3, column I), is much higher than that calculated by *Kopal* [1966] (Table 3, column II), who apparently used an incorrect value of D_f (Table 2). The present result of 1.99 is far in excess of the 0.80 expected from Baldwin's observed V_r/V_t data in Figure 3, and the 0.4–0.8 range observed for terrestrial explosion craters. Despite the deceptively youthful morphology of Tycho, its existing surface geometry does not appear to be that of a freshly formed impact crater.

The impact configuration of Tycho predicted by the author's model is shown in Figure 5. The resulting volumes appear in Table 3, columns III and IV. Column III presents the volumes obtained from an observed D_0 of 128 km. In column IV, a D_0 of 123 km (equation 5) compensates partially for the omission of a

TABLE 2. Present (Observed) and Original (Predicted) Configurations of the Lunar Crater Tycho (km)

Source	D_r	D_0	D_f	D_t	R_i	R_a	R_t
MacDonald [1931], present configuration	87.0	...	...	...	4.65	2.40	...
Baldwin [1963], present configuration	86.9	127.5	...	...	4.27	2.41	...
Baldwin [1963], original impact shape	86.9	127.5	0	75.0	6.60	2.33	4.27
Kopal et al. [1965], present configuration	...	...	48.0	...	5.40	...	...
Kopal [1966], present configuration	88.0	130.0	72.0	79.5	4.60	2.40	2.20
This study, present configuration	88.0	128.0	46.0	66.0	4.60	2.40	2.20
This study, original impact shape	88.0	128.0 123.0	0	76.8	10.90	3.38	7.52

TABLE 3. Observed (I, II) and Predicted (III–V) Volumes ($\times 10^3$ km^3) of the Lunar Crater Tycho Volumes defined in Table 1 and Figure 2.

	Observed Configuration		Predicted Impact Configuration		
	(Pike) I	(Kopal) II	(Pike) III	(Pike) IV	(Baldwin) V
V_a	...	...	36.76	36.76	21.80
V_1	22.10	22.75	31.26	29.82	21.65
V_2	11.08	13.25	18.05	18.05	12.20
V_r	11.02	9.50	13.21	11.77	9.45
V_i	5.48	9.90*	18.71	18.71	9.60
V_e	−5.64	+0.40	+5.50	+6.94	+0.15
V_r/V_i	1.99	0.96	0.71	0.63	0.98

* Incorrectly given as 8.90 by *Kopal* [1966].

correction to the quantity V_2 for observed concavity of both inner and outer rim slopes of Tycho. The respective V_r/V_i values, 0.71 and 0.63, are entirely consistent with an explosive impact origin for Tycho. They fall well within the experimentally derived range of 0.4–0.8, and they also fit a linear extrapolation of the small fresh craters in Figure 3.

The fresh impact crater model yields a V_e range of 5.50 to 6.94 $\times$ 10^3 km^3 for Tycho. Perhaps 0.40 to 0.75 of this volume actually may be attributed to extra-rim ejecta. Structural impact deformation of the bedrock beneath the crater rim accounts for most of the remaining fraction [*Carlson and Jones*, 1965]. Some independent calculations by Z. Kopal approximate these results. *Kopal* [1966] has extrapolated the mass of ejecta believed necessary to produce all small 'secondary' craters observed in the areas of Mare Cognitum that are delimited by the bright rays of Tycho. He obtains a total *minimum* volume of material ejected beyond the outer rim slope of Tycho. His result, 5.1 to 9.3 $\times$ 10^9 km^3, exceeds that obtained by a very different method in the present investigation only by a factor of 2. *Marcus* [1966] finds the frequency of all small craters on Tycho's rays to exceed by 15 times the predicted frequency of terrestrial explosion crater secondaries. In addition, neither Kopal nor Marcus attempted to distinguish Tycho ray secondaries from subjacent pre-Tycho secondaries and small impact primaries. Thus the small craters requiring explanation by secondary (and tertiary) impact are far fewer than the number assumed by Kopal. Consequent adjustment of Kopal's volumes should bring them into even closer agreement with the results of the present model.

Baldwin [1963] also has derived a model for fresh impact crater dimensions. Table 2 lists the R_i and R_e values predicted for Tycho; and column V of Table 3, the resulting volumes. The large number of relatively shallow modified lunar craters, such as Tycho, included in Baldwin's $R_i{:}D_r$ and $R_e{:}D_r$ graphs have lowered the slopes of his predictive expressions. The resulting R_i and R_e values and volumes are too low. His V_r/V_i value for Tycho is only 0.98. It is well above the 0.4–0.8 range expected for freshly formed explosive impact craters. The allometric model derived in the present study predicts a more probable original impact surface geometry for Tycho. It should apply to all other lunar impact craters as well.

Post Impact Processes

Geological explanations of differences between fresh and deformed impact craters are few. This author favors an interpretation based on the principle of isostasy [*MacDonald*, 1931; *Baldwin*, 1963; *Masursky*, 1964]. An explosion crater is in equilibrium with its genetic environment only briefly. Upon dissipation of the explosion, physical conditions revert to those of the nonimpact planetary surface environment. The shape of the crater will tend toward equilibrium with the new environment, possibly resulting in a new, nonallometric surface geometry. The mass contained in the rim of a crater much over 10 km D_r constitutes a severe loading of the moon's outer layers. In the absence of lunar surface erosional processes capable of reducing this mass excess, the substrata yield to the imposed stresses and deform viscoelastically. The annular mountain range sinks until isostasy is achieved. Similarly, the floor of a large crater rises to reduce the mass deficiency of the original inner crater depression.

Both theory [*Daneš*, 1962] and observation (Table 2) indicate that the floor of a large crater rises more rapidly than its rim subsides. V_e, the initial intrinsic excess of V_i over V_r, diminishes until eventually the relationship may be reversed. The 1:1 correspondence observed originally by Schroeter for some larger lunar craters is only a transitional condition in which

the two volumes happen to be equal. The great variety of V_r/V_t values observed for lunar craters over 10–20 km D_r may be interpreted in terms of the relative completeness of isostatic compensation. The rate of adjustment undoubtedly is some function of crater size (D_r) and absolute age (t) [*Daneš*, 1962].

At least three other phenomena may be related to the 10–20 km D_r threshold separating fresh from modified explosive impact craters. First, central peaks do not occur frequently in craters much below 15 km D_r. Second, rugged arcuate structural and slump terraces exist only on the rims of craters over 20–25 km D_r. Although the post-impact genesis of these two features can not be demonstrated conclusively, Orbiter 2 pictures show that volcanic activity probably has contributed to the central peaks of the familiar rayed crater Copernicus.

On the other hand, the work of *MacDonald* [1931], *Warner* [1961*b*], *Daneš* [1962], *Baldwin* [1963], and *Masursky* [1964] suggests strongly that the flat floors so characteristic of Tycho, Copernicus, and other large lunar craters are acquired subsequently to initial formation of the craters. Freshly formed explosion craters do not have flat floors. The scarcity of floors in lunar craters under 10 km D_r indicates that they are not characteristic of unaltered lunar craters. Flat floors suggest hydrostatic filling of a bowl-shaped depression. The floors of lunar craters may comprise series of lava flows extruded into crater depressions through the subjacent impact breccia lens as the craters underwent isostatic adjustment. Possibly, these floor-forming lavas were derived from material mobilized beneath the craters by isostatically generated stresses. The complex lithologies encountered in some shallow and apparently resprung terrestrial meteorite crater floors may reflect operation of similar processes on earth [*Masursky*, 1964].

Prolonged and involved endogenetic alteration of a large impact crater can yield a surface morphology deceptively unlike the original impact crater shape. It is not surprising that some authorities choose to interpret the configurations of large lunar craters as surface cauldrons and large calderas rather than as deformed impact structures [*Green*, 1965, *McCall*, 1966, *Shteinberg*, 1965]. However, large lunar craters photographed by Ranger and Orbiter spacecraft do not appear to be of totally internal genesis. Rather they exhibit an explosive impact morphology modified by subsequent internal activity. The morphology and morphometry of the moon's craters require an explanation commensurate with their observed complexity. This investigation has developed one alternative to the venerable but illusory Schroeter's rule and its prevalent oversimplified impact interpretation.

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