Aspect ratios and magma overpressures of non-feeder dikes observed in the Miyake-jima volcano (Japan), and fracture toughness of its upper part

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[1] We present a new method for estimating the length and maximum thickness (aperture) of a dike from the observed opening at one dike tip. We apply the method to 15 arrested non-feeder dikes (where the upper tip is known, the lower tip unknown) in the caldera walls of Miyake-jima, Japan, to estimate the length-thickness ratio, as well as the magma overpressure and fracture toughness. The calculated length-thickness ratio ranges from 61 to 246, with an average of 136. The ratios are low because the dikes are emplaced close to the surface in comparatively compliant (soft) rocks. Using these ratios and the appropriate elastic constants, the calculated magmatic overpressures of the dikes are between 2.3 and 8.9 MPa, and the stress intensity factors between 38 and 117 MPa $m^{1/2}$. All these values are within the range of typical in situ estimates, supporting the validity of this new method. **Citation:** Kusumoto S., N. Geshi, and A. Gudmundsson (2013), Aspect ratios and magma overpressures of non-feeder dikes observed in the Miyake-jima volcano (Japan), and fracture toughness of its upper part, *Geophys. Res. Lett.*, *40*, 1065–1068, doi:10.1002/grl.50284.

1. Introduction

[2] The Miyake-jima volcano is a basaltic-andesitic stratovolcano located at the volcanic front of the Izu-Mariana subduction zone and is one of the most active volcanoes in Japan (Figure 1). Eruptions in the past 600 years have occurred at intervals of 21 to 69 years [e.g., *Miyazaki*, 1984; *Tsukui and Suzuki*, 1998; *Geshi et al*., 2002]. The caldera that formed at the summit of Miyake-jima in 2000 has a diameter of 1.7 km and a depth of 450 m [e.g., *Geshi et al*., 2002; *Geshi*, 2009]. Many feeder and non-feeder dikes are exposed in the walls of the caldera [*Geshi et al*., 2010]. *Geshi et al*. [2010] found systematic differences between the shapes of the feeder and the non-feeder dikes and suggest that these differences are primarily related to the free surface affecting the upper geometry of the feeder dikes. Using a finite-element model, *Geshi et al*. [2012] also show that the thickness variations of non-feeder dikes depend on abrupt changes in stiffness among layers and relate also to the volumetric flow rate of the magma.

[3] In all these considerations, it is important to know the overpressure of the magma in the dikes. Here overpressure is defined as the total magmatic pressure in the dike minus the minimum principal compressive (maximum tensile) stress σ_3 acting on the dike walls. Many have estimated the magma overpressure based on dike thickness (or aperture for a magma-filled dike fracture) and the assumption that the host rock behaves as elastic [e.g., *Delaney and Pollard*, 1981; *Pollard et al*., 1983; *Pollard and Segall*, 1987]. In such studies, the following fundamental equation [e.g., *Sneddon and Lowengrub*, 1969] is used:

$$u_y = \frac{2(1-\nu^2)p_0}{E}\sqrt{a^2 - x^2}, \tag{1}$$

where E, ν, a, and p_0 are, in that order, Young's modulus and Poisson's ratio of the host rock, the half-length of the dike, and the magma overpressure in the dike. Also, x is the coordinate of the observation point used in the measurement of the opening displacement of the dike-fracture wall u_y and is measured from the center toward the tip of the dike (Figure 2). Equation (1) is valid for internal magma overpressure p_0 acting on a line crack in an infinite elastic body and also for an extension fracture in an infinite elastic body subject to an external tensile stress (p_0 is then the magnitude of the tensile stress). Here equation (1) gives the crack shape formed by the overpressure and/or external stress. For example, the magnitude of the regional tensile stress field has been estimated from the aspect (length/aperture) ratios of tension fractures in the rift zone of Iceland [*Gudmundsson*, 1983]. Since equation (1) is very simple and suggests a linear relation between the crack shape and the driving pressure or stress, it has also been applied to estimate the overpressure associated with observed dikes in the field, although more general solutions exist [e.g., *Gudmundsson*, 2011; *Gudmundsson et al*., 2012; *Kusumoto et al*., 2013].

[4] When using equation (1), we need to know the total length of the dike and the elastic constants of the host rock. We can measure the elastic constants of the host rock or estimate their typical values from tables in textbooks and/or data handbooks [e.g., *Bell*, 2000; *Lama and Vutukuri*, 1978]. However, in general, it is difficult to determine the center and/or the total length of a dike since only parts of most dikes can be observed and measured in the field [e.g., *Gudmundsson*, 1984; *Gudmundsson et al*., 2012]. Length, in this sense, has two meanings: the dip dimension for dikes seen in vertical sections and the strike dimension for dikes seen in lateral sections (plan view). The intended meaning is normally clear from the context, but defined where necessary.

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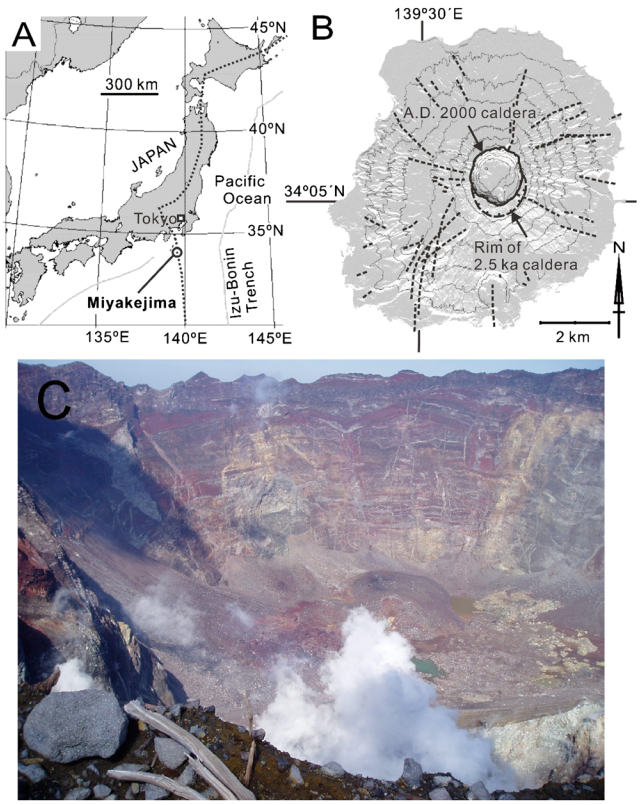

Figure 1. Location of the study area. (a) Location map of Miyake-jima volcano. The broken line shows the location of the volcanic front. (b) Outline of the caldera formed during the 2000 collapse and eruption and radial trends of major eruption fissures (broken lines after *Tsukui et al.* [2005]). The arrows indicate the rim of each caldera and do not indicate the regional stress direction. (c) Distant view of the northwestern part of the caldera wall. There is a swarm of non-feeder dikes dissecting the frontal wall.

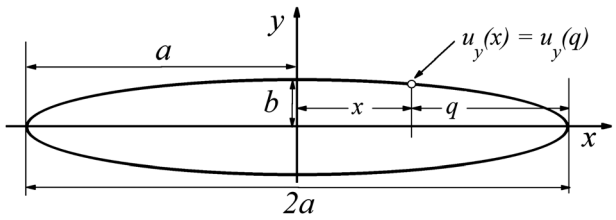

Figure 2. Basic crack model and relationships among the parameters used. The crack model is based on plane-strain assumption, and the crack is regarded as infinite in the direction perpendicular to the *x*-*y* plane. Parameters a and b are the half-length and half maximum thickness of the dike, respectively. The parameter q provides the location of the observation point for the opening displacement, u_y, of the dike fracture, and the location of the observation point is measured from the dike tip. There exists the relationship $a = x + q$ between q and x (location of the observation point) as measured from the center of the dike.

[5] In the present paper, the origin of the coordinate system is not at the center of the dike but rather at its tip. For this purpose, we introduce a new parameter, q, related to the location of the origin (Figure 2). The opening fracture displacement $u_y(x)$ at a given location x is thus expressed by $u_y(q)$ and q, respectively. We propose a new method for estimating the total length (strike or dip dimension) and the maximum thickness of the dike (or dike segment) using the values of q and $u_y(q)$. We apply the new method to data on non-feeder dikes from the caldera walls of Miyake-jima in Japan and estimate the dike aspect ratios, magma overpressures, and fracture toughness.

2. Method

[6] We can rewrite equation (1) as the equation of an ellipse with a semi-major axis a as follows:

$$\frac{x^2}{a^2} + \frac{u_y^2}{\left[\frac{2(1-\nu^2)p_0}{E}\right]^2 a^2} = 1. \tag{2}$$

[7] By introducing the parameter b as the semi-minor axis of the ellipse, we obtain the following:

$$b = \frac{2(1-\nu^2)p_0}{E} a. \tag{3}$$

[8] This equation gives the half maximum thickness of the dike b, which is equal to the opening displacement u_y at $x = 0$ in equation (1). By rewriting equation (2) in terms of equation (3), we obtain the following:

$$\frac{x^2}{a^2} + \frac{u_y^2}{b^2} = 1. \tag{4}$$

[9] If the distance from the dike tip to the point of observation along the fracture is q, then the relationship $x + q = a$ exists between x and the semi-major axis (a) of the dike (Figure 2). By transforming this relationship into $x = a - q$ and substituting this expression into equation (4), we obtain the following relation:

$$u_y(q) = \frac{b}{a}\sqrt{q(2a-q)}. \tag{5}$$

[10] If q is 0 or $2a$, then u_y is 0; if q is a, then u_y is the half maximum thickness, i.e., b. From equation (5), we can estimate both the half-length (a) and the half maximum thickness (b) of a dike from the opening displacement, $u_y(q_i)$, where q_i ($0 \le q_i \le 2a$), observed at the dike tip. For these estimates, we must use a nonlinear least squares method (e.g., the Gauss-Newton method [*Aster et al.*, 2005], which is used here, or the Levenberg-Marquardt method [*Aster et al.*, 2005] alternatively) because the parameter a is nonlinear in equation (5).

3. Application to Dikes Observed in Miyake-jima, Japan

[11] We applied equation (5) to 15 non-feeder dikes studied by *Geshi et al.* [2010] in the caldera walls of Miyake-jima and estimated the half-lengths (half dip dimensions) and half maximum thicknesses of the dikes. As examples, we show both the observed data and the optimum dike shapes estimated by the Gauss-Newton least squares method in Figure 3.

[12] The estimated half-lengths (a) and half maximum thicknesses (b) of dikes are shown in Table 1. The estimated half-lengths (a) range from 43.3 to 117.0 m, the average

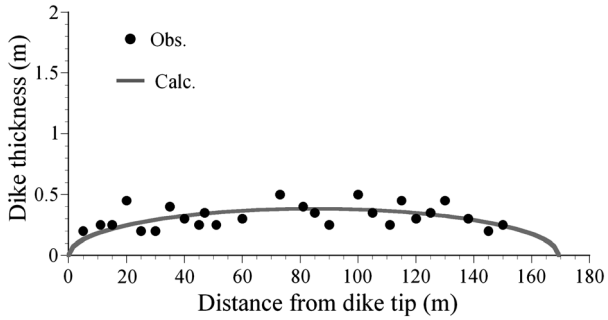

Figure 3. Comparison between the observed dike shape (78-F) and the theoretical dike shape. The black circles indicate observed data, and the gray line indicates the theoretical dike shape as calculated from the magma overpressure estimated by the least squares method based on equation (5). There is no data far from the point at 150 m from the dike tip. The estimated half length (a) of the dike is 84.7 m.

dike half-length (half dip dimension) being 73.3 m. Similarly, the estimated half maximum thicknesses of the dikes (b) range from 0.3 to 1.2 m, the average being 0.6 m.

[13] We define the aspect ratio of the dikes as the maximum half-length (a)/half maximum thickness (b) and calculate the aspect ratios from the data on the 15 dikes. The estimated aspect ratios of the dikes are from 61 to 246, with an average of 136 (Table 1).

[14] We also rewrite equation (3) as follows:

$$p_0 = \frac{E}{2(1-\nu^2)}\left(\frac{a}{b}\right)^{-1}. \tag{6}$$

[15] This equation can be used to estimate the magma overpressure, p_0, using appropriate values for the host rock Young's modulus and Poisson's ratio. Since dikes observed in Miyake-jima are located near the surface and dissect mostly soft pyroclastic layers, we assume the low Young's modulus (E) of 1 GPa as well as a Poisson's ratio (ν) of 0.25 for the host rock [e.g., *Bell*, 2000; *Geshi et al*., 2010]. From the aspect ratios and these elastic parameters, we obtained the magma overpressure values in Figure 4 and Table 1. The estimated magma overpressure ranges from 2.3 to 8.9 MPa, with an average of 4.8 MPa.

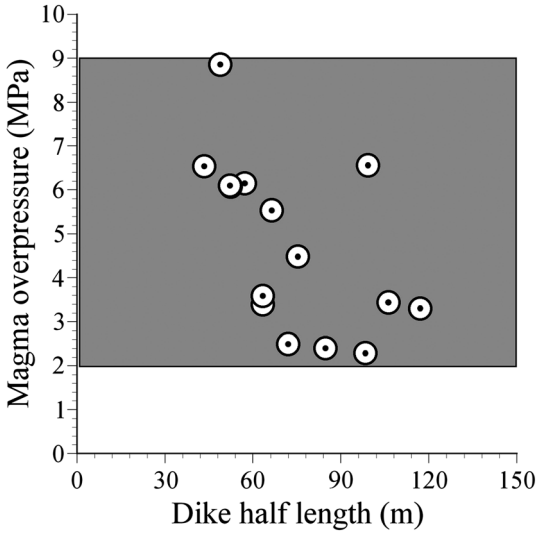

Figure 4. Relationship between dike length (a) and magma overpressure. The estimated magnitudes of the overpressure range from 2.3 to 8.9 MPa. There is a weak negative correlation between the dike length and the overpressure.

[16] In addition, we calculated the stress intensity factor, K_I, using the following equation [e.g., *Sneddon and Lowengrub*, 1969; *Anderson*, 2005]:

$$K_I = p_0\sqrt{\pi a}. \tag{7}$$

[17] The estimated stress intensity factors associated with the 15 dikes vary from 38 to 117 MPa $m^{1/2}$, with an average of about 69 MPa $m^{1/2}$ (Table 1). For a fracture to propagate, the stress intensity factor must reach a certain minimum critical value defined as the fracture toughness, while if the stress intensity factor of propagating fracture is less than the fracture toughness, the fracture becomes arrested [e.g., *Gudmundsson*, 2011].

4. Discussion

[18] *Gudmundsson* [1984] measured the aspect ratios of 12 Tertiary dikes in northwestern Iceland as between 300

Table 1. Half-length (a), Half Maximum Thickness (b), Aspect Ratio (a/b), Magma Overpressure, and Stress Intensity Factors of Dikes Observed on the Caldera Wall in Miyake-jima, Japan[a]

Name of Dike	Half-length (m)	Half Maximum Thickness (m)	Aspect Ratio a/b	Overpressure (MPa)	Stress Intensity Factor (MPa $m^{1/2}$)
78-F	84.7	0.4	212	2.4	39
88-A	63.3	0.4	158	3.4	48
88-B	63.3	0.4	158	3.6	51
88-D	66.4	0.7	95	5.5	79
88-F	72.0	0.3	240	2.5	38
88-G	75.3	0.6	126	4.5	69
90-02	43.3	0.5	87	6.5	76
90-03	57.1	0.7	82	6.2	83
90-04	117.0	0.7	167	3.3	63
94-01	106.1	0.7	152	3.4	62
94-02	48.8	0.8	61	8.9	110
94-04	52.4	0.6	87	6.1	78
94-05	52.1	0.6	87	6.1	78
98-B	99.1	1.2	83	6.6	117
270-2	98.3	0.4	246	2.3	40
Average	73.3	0.6	136	4.8	69

[a]Names of dikes correspond to names given in *Geshi et al*. [2010].

to 1500, with an average of about 800. Since the average aspect ratio of the dikes in Miyake-jima is 136, it means that the aspect ratio of the dikes in Miyake-jima is about one sixth of the aspect ratio of the dikes in northwestern Iceland. The main reason for this difference is the difference in magma intrusion environment, in particular the mechanical properties of the host rock. The dikes in Iceland are exposed at a depth of 500–1000 m below the surface at the time of dike emplacement and dissect mostly comparatively stiff (Young's modulus of 5–10 GPa) basaltic lava flows. In contrast, the dikes in Miyake-jima are exposed at very shallow depths and dissect mostly soft pyroclastic layers.

[19] The magnitudes of the overpressure estimated in the present study, 2.3–8.9 MPa, are similar to those obtained in many other studies [e.g., *Delaney and Pollard*, 1981; *Poland et al*., 2008; *Geshi et al*., 2010]. The overpressure values here are partly the results of the estimated low Young's modulus of the host rock. All the estimated magma overpressures are less than the maximum in situ tensile strength of rocks (9 MPa [e.g., *Gudmundsson*, 2011]), indicating that the assumed Young's modulus is reasonable for the uppermost layers of Miyake-jima. In the future, these methods may help put limits on the minimum magma overpressure necessary for a dike to have the chance of erupting during an unrest period.

[20] The estimated stress intensity factors (38 to 117 MPa $m^{1/2}$; average, 69 MPa $m^{1/2}$) fall within the range of previous field or in situ values (30 to 150 MPa $m^{1/2}$), as obtained in many studies [e.g., *Delaney and Pollard*, 1981; *Rubin and Pollard*, 1987; *Parfitt*, 1991; *Rivalta and Dahm*, 2006; *Gudmundsson*, 2009]. Thus, the estimated stress intensity factor may be regarded as reliable and appropriate for the fracture toughness of the uppermost layers of Miyake-jima because dikes are arrested.

5. Conclusions

[21] In the present study, we propose a new method for estimating the length (strike or dip dimension) and half maximum thickness of dikes based on the measured dike-opening displacement close to the dike tip. The proposed method is very useful because, in general, it is difficult to determine the total length of a dike (or dike segment) in the field, while this information is needed in order to estimate the magma overpressure from the dike shape. We applied the new method to 15 non-feeder dikes observed in the caldera walls of Miyake-jima and obtained dike half-lengths from 43.3 to 117.0 m and dike half maximum thicknesses from 0.3 to 1.2 m. The aspect ratios of the dikes are from 61 to 246 and about one sixth of typical aspect ratios of Tertiary dikes in Iceland. The reason for this difference is the much stiffer host rocks (at greater depths) for the dikes in Iceland than for those in Miyake-jima. The estimated overpressures for the Miyake-jima dikes are from 2.3 to 8.9 MPa, and the stress intensity factors from 38 to 117 MPa $m^{1/2}$. The magma overpressures of the non-feeder dikes do not exceed the maximum in situ tensile strength of rocks (9 MPa) so that the host rock Young's modulus used (1 GPa) may be regarded as reasonable. In addition, the calculated stress intensity factors are similar to other in situ estimates and are likely to be representative for the uppermost of the Miyake-jima volcano.

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