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Explosive Volcanic Eruptions – II
The Atmospheric Trajectories of Pyroclasts

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Summary

The formulation and solution of the general equations of motion of pyroclasts in the Earth's atmosphere are described. Ranges of particles ejected from a vent, and times of fall of particles released from an eruption column or ash cloud, are computed for those values of particle radius (0·01 to 100 cm), density (0·5 to 3·5 g cm^{-3}), launch velocity (10 to 1000 m s^{-1}), elevation (45, 66 and 87 degrees) and release height (0 to 50 km) of practical importance. The values so obtained are presented in tabular and graphical form, and their use in determining the physical parameters of an explosive eruption is described.

1. Introduction

In the first paper in this series (Walker, Wilson & Bowell 1971, hereinafter referred to as Paper I), the terminal velocities at sea level were calculated for wide ranges of diameter and density of solid particles, using three simple particle shapes, classified as (i) smooth spheres, (ii) rough spheres, and (iii) cylinders. Terminal velocities were measured for pumice and lithic fragments, and sanidine crystals, and compared with the corresponding calculated values. It was found that these particles, all being representative examples of pyroclastic débris, behaved aerodynamically as cylinders. With this essential identification of effective particle shape it is possible, using appropriate experimental data on aerodynamic drag forces, to compute the trajectories of pyroclastic fragments under almost all circumstances of eruption and subsequent movement in the atmosphere. This paper describes the general numerical solution of the equations of motion of pyroclastic particles.

The methods of determining physical parameters of explosive eruptions are discussed. It is shown that the most important calculable quantities required are (a) the ranges in air of particles ejected at various velocities from a vent, and (b) the times of fall of particles released at various heights from an eruption column and cloud. Values are presented for these quantities, using wide ranges of particle size, density, launch velocity and release height.

As an example of the use of particle ranges, the values deduced by Fudali & Melson (1972) for the launch velocities of fragments ejected explosively from the Arenal volcano (at least 600 m s^{-1}) are shown to be too small due to the neglect of compressibility effects at particle velocities near Mach number 1·0. After a re-discussion of the analysis, revised values of at least 660 m s^{-1} are found.

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2. Characteristics and analysis of explosive eruptions

In an explosive eruption, especially of the Plinian variety (Walker & Croasdale 1970), it is generally found that smaller particles are carried up to considerable heights by the eruptive column and subsequently released from this column or from the ash cloud in which it terminates. Larger blocks are launched more nearly ballistically from the vent, since as particle diameter increases, the ratio of volume to cross-sectional area also increases, and aerodynamic forces are dominated by gravity. Sufficiently large blocks, may therefore, decouple rapidly from the high-velocity gas phase of the eruption column and are also only slightly affected by the presence of the atmosphere.

The most important physical parameters of an explosive eruption are (1) the velocity of blocks leaving the vent; (2) the velocity of the eruption column leaving the vent; (3) the height and shape of the eruption column and ash cloud; (4) the mass and temperature of the gaseous component of the eruption column; (5) the total mass of pyroclastic material erupted and its aerial distribution around the vent. Items (2) and (4) could be used to obtain an estimate of the thermal energy carried by the gas phase and could, in principle, be obtained from (3) on solution of the appropriate hydrodynamic equations; this will be the subject of future study. The present paper is concerned with items (1), (3) and (5).

Items (1) and (5) together lead to an estimate of the kinetic energy carried by larger blocks. In practice, it is rarely possible to make measurements of velocities of ejecta during actual explosive eruptions, although this may be done if the explosions are of sufficiently small magnitude (Guest 1970, private discussion and unpublished data). There is, however, a considerable body of data on the aerial distribution of pyroclastic débris (Walker & Croasdale 1970) and it is appropriate to make the maximum use of this data. The principal field measurement is the final position of particles of a given size, i.e. the range of the particles. It is necessary, therefore, to assume various values for the speeds and elevations of larger blocks ejected from a vent and compute the ranges of such blocks. Comparison of the computed ranges with the observed ranges of blocks of different sizes then permits the identification of those values of speed and elevation that actually occurred in the eruption. Accordingly, ranges have been calculated for particles with radii between 0·01 cm and 100 cm and densities of 0·5, 1·0, 2·5, and 3·5 g cm^{-3} launched at speeds of 10, 30, 100, 300, 600 and 1000 m s^{-1} and at elevations of 45, 66 and 87 degrees.

It is possible to make some progress towards obtaining item (3) from the aerial distribution of débris. On a windless day, an eruption column will release particles of various sizes at different heights and, as it expands with height, at different distances from the vent. These particles will settle to the ground and form an azimuthally symmetrical distribution, with smaller particles found at greater distances from the vent. If, however, an appreciable wind exists, as is usually the case, the settling particles will be carried down-wind forming an elongated distribution, the major axis defining the wind direction. Under these circumstances, the horizontal displacement of particles will depend on (a) the wind speed and (b) the time of fall. Once again, it will be necessary to assume a model of the eruption column, and a value for the wind speed, and find the calculated distribution pattern that most closely approximates the observed distribution. The principal quantity required, the time of fall of particles, is given here for particles of radii 0·01 to 10 cm and densities 0·5, 1·0, 2·5 and 3·5 g cm^{-3} falling from heights up to 50 km. These values should cover the entire range of interest (Walker 1969, private discussions).

3. Formulation of the problem

Let v be the speed at time t of a particle of mass m currently at position (x, y)

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moving along a path inclined at angle θ to the x-axis (horizontal) and experiencing a total drag force $W(v)$. It is justifiable, in the case of pyroclastic particles, to neglect lift forces, as noted in Paper I. The acceleration due to gravity is $-g(y)$ in the positive y-direction (vertically upwards) and the origin of co-ordinates coincides with the Earth's surface, the curvature of the Earth being neglected. Let us define:

$$z = v\cos\theta \tag{1}$$

$$e = v\sin\theta \tag{2}$$

$$F(v) = W(v)/mv. \tag{3}$$

The motion is then defined by the solutions of

$$\frac{dz}{dt} = -zF(v) \tag{4}$$

$$\frac{de}{dt} = -eF(v) - g \tag{5}$$

$$\frac{dx}{dt} = z \tag{6}$$

$$\frac{dy}{dt} = e, \tag{7}$$

subject to

$$v = (z^2 + e^2)^{\frac{1}{2}}. \tag{8}$$

$W(v)$ depends in general on the shape of the particle as well as on v, and also on properties of the Earth's atmosphere. There is, however, a considerable body of experimental data on drag phenomena (Hoerner 1965) and it is well established that the problem is greatly simplified by defining the drag coefficient, C, and Reynolds number, R, thus:

$$C = \frac{2W(v)}{v^2\,\pi a^2\,\rho(x, y)} \tag{9}$$

$$R = 2av\rho(x, y)\,v(x, y), \tag{10}$$

where ρ and v are the density and viscosity of air, both in general being a function of position, and a is a typical radius of the particle. With these definitions, C can be expressed as a function of R alone, for any one particle shape, for a wide range of all the parameters as long as v is rather less than the local speed of sound, i.e. as long as M, the Mach number, is less than about 0·7. Hoerner (1965) gives (graphically) the function $C(R)$ for several basic particle shapes, together with correction factors $K(M)$ for values of M up to at least 5. We are specifically concerned here with $C(R)$ and $K(M)$ for cylindrical objects.

For the present purpose it has been assumed that ρ varies only with height, and that v varies only with absolute temperature, T, in turn related uniquely to height. The variations of ρ and T have been taken from Allen (1963) and v obtained from T using the formula given by Sutherland (1893) for air:

$$v = 0{\cdot}000172\left(\frac{390}{T+117}\right)\left(\frac{T}{273}\right)^{1.5}. \tag{11}$$

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Since $C(R)$, $K(M)$, $\rho(y)$ and $T(y)$ are available only graphically or at discrete points, and are required here as continuous functions, they have been divided into suitable sections and approximated by polynomials. The polynomial representations in adjacent sections are matched to at least seven decimal places to ensure effective continuity of the function. The ranges of the independent variables for which these polynomials are valid are: $10^{-2} \leqslant R \leqslant 10^{7}$; $0 \leqslant M \leqslant 5{\cdot}0$; $0 \leqslant y \leqslant 50$ km. By means of equations (9), (10) and (11), and the polynomials $C(R)$, $K(M)$, $\rho(y)$ and $T(y)$, therefore, $W(v)$ can be found for any values of v and y.

Finally, $g(y)$ can be calculated from

$$g(y) = \frac{g_0}{\left(1+\frac{y}{E}\right)^2}, \tag{12}$$

where g_0 is the gravitational acceleration at $y = 0$ and E is the radius of the Earth.

4. Solution of the equations of motion

Having specified the force terms $W(v)$ and $g(y)$ we now return to the solution of equations (4) to (7). A Runge–Kutta method can be used as follows (Kooy & Uytenbogaart 1946). Let increments D_1, D_2, D_3, D_4, occur in x, y, z and e as a result of an increment τ in t. Then it can be shown that

$$D_i = R_{i1}/6 + R_{i2}/3 + R_{i3}/3 + R_{i4}/6, \qquad i = 1, 2, 3, 4 \tag{13}$$

where

$$\left.\begin{aligned} R_{11} &= z\tau; \qquad R_{21} = e\tau \\ R_{31} &= -F\left(\sqrt{(z^2+e^2)}\right)z\tau \\ R_{41} &= -F\left(\sqrt{(z^2+e^2)}\right)e\tau - g\tau \end{aligned}\right\} \tag{14a}$$

$$\left.\begin{aligned} R_{12} &= (z+\tfrac{1}{2}R_{31})\,\tau; \qquad R_{22} = (e+\tfrac{1}{2}R_{41})\,\tau \\ R_{32} &= -F\left(\sqrt{((z+\tfrac{1}{2}R_{31})^2+(e+\tfrac{1}{2}R_{41})^2)}\right)(z+\tfrac{1}{2}R_{31})\,\tau \\ R_{42} &= -F\left(\sqrt{((z+\tfrac{1}{2}R_{31})^2+(e+\tfrac{1}{2}R_{41})^2)}\right)(e+\tfrac{1}{2}R_{41})\,\tau - g\tau \end{aligned}\right\} \tag{14b}$$

$$\left.\begin{aligned} R_{13} &= (z+\tfrac{1}{2}R_{32})\,\tau; \qquad R_{23} = (e+\tfrac{1}{2}R_{42})\,\tau \\ R_{33} &= -F\left(\sqrt{((z+\tfrac{1}{2}R_{32})^2+(e+\tfrac{1}{2}R_{42})^2)}\right)(z+\tfrac{1}{2}R_{32})\,\tau \\ R_{43} &= -F\left(\sqrt{((z+\tfrac{1}{2}R_{32})^2+(e+\tfrac{1}{2}R_{42})^2)}\right)(e+\tfrac{1}{2}R_{42})\,\tau - g\tau \end{aligned}\right\} \tag{14c}$$

$$\left.\begin{aligned} R_{14} &= (z+R_{33})\,\tau; \qquad R_{24} = (e+R_{43})\,\tau \\ R_{34} &= -F\left(\sqrt{((z+R_{33})^2+(e+R_{43})^2)}\right)(z+R_{33})\,\tau \\ R_{44} &= -F\left(\sqrt{((z+R_{33})^2+(e+R_{43})^2)}\right)(e+R_{43})\,\tau - g\tau. \end{aligned}\right\} \tag{14d}$$

For any initial set of values x_0, y_0, v_0, θ_0 and t_0, values of z_0 and e_0 can be found from equations (1) and (2). Successive increments D_i can be obtained from the above set of equations (14) using a suitable time step τ until any specified criterion applies—for example until the particle reaches the ground. Furthermore, since the value of τ may be changed between steps, τ can be decreased when large velocities or accelerations occur, to maintain accuracy, or increased when only small positional changes are occurring, to minimize the amount of calculation needed.

In practice, values of τ leading to 1 per cent computational accuracy in all the final parameters have been used. This means that the true accuracy is limited by

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(a) the extent to which real pyroclasts behave as cylinders; (b) the accuracy with which $C(R)$ and $K(M)$ are determined by the experimenters. Both of these quantities are hard to estimate. It was found in Paper I that computed terminal velocities of small particles agreed with experimental values to within 10 per cent (often to within 5 per cent). It would be realistic, therefore, to estimate that parameters presented here for the smallest particles are within 10 per cent of the true values. The accuracy of the parameters of larger particles will be progressively greater, however, as the ratio of gravitational forces to drag forces rises.

The above method of solution of the equations of motion has been programmed in FORTRAN. When implemented on the University of Lancaster's I.C.L. 1905F, the program required 5056 core locations. The computation of the trajectory of a 0·01-cm radius particle launched at high speed near the ground (the worst case), required 10 s; the time of fall of such a particle from a height of 50 km could be obtained in 20 s.

5. Calculated ranges and times of fall

Table 1 presents the logarithms (to base 10) of the ranges, in centimetres, of particles having radii between 0·01 and 100 cm and launched at speeds between 10 and 1000 m s^{-1} with an elevation of 45 deg. The values corresponding to the four densities 0·5, 1·0, 2·5 and 3·5 g cm^{-3} are given. Tables 2 and 3 supply the corresponding range logarithms for elevations of 66 and 87 degrees respectively. Values are omitted from these tables where (a) they are unlikely to be needed in practice (very small particles launched at great speeds), and (b) they are not essential for accurate interpolation between listed range logarithms.

Figures 1, 2, 3 and 4 present the fall times of particles with densities 0·5, 1·0, 2·5 and 3·5 g cm^{-3} respectively, as a function of particle radius. In each case five curves appear, corresponding to release heights of 5, 10, 20, 30 and 50 km.

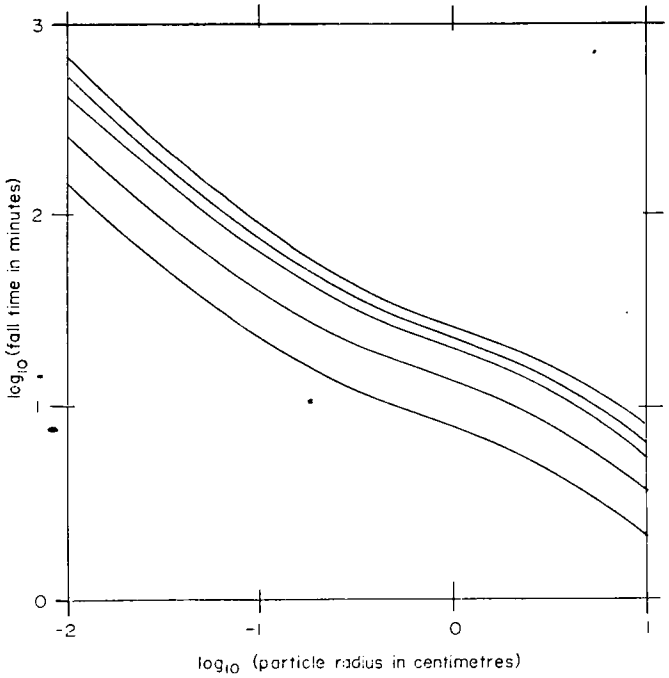

FIG. 1. Fall times of particles having density 0·5 g cm^{-3}. The curves (from top to bottom) are for particles released from heights of 50, 30, 20, 10 and 5 km.

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Table 1

Logarithms (to base 10) of ranges (in centimetres) of particles launched at 45 deg.

Velocity ($m\ s^{-1}$)	Density ($g\ cm^{-3}$)	Radius in centimetres 0·01	0·1	0·3	1·0	3·0	10·0	30·0	100·0	log (vacuum range, cm)
	3·5	1·9034	2·7439		2·9556		3·0032		3·0075	
10	2·5	1·7745	2·6772		2·9369		3·0017		3·0070	
	1·0	1·4089	2·4490		2·8559		2·9914		3·0060	3·0088
	0·5	1·1222	2·2380		2·7579		2·9760		3·0043	
	3·5	2·1256	3·1424		3·6440		3·9315		3·9583	
30	2·5	1·9872	3·0324		3·5719		3·9208		3·9571	
	1·0	1·6042	2·7128		3·3383		3·8680		3·9483	3·9630
	0·5	1·3105	2·4556		3·1364		3·7965		3·9356	
	3·5	2·3078	3·3201		4·0418		4·6867		4·9660	
100	2·5	2·1661	3·1106		3·7372		4·6173		4·9514	
	1·0	1·7773	2·8461		3·6042		4·3999		4·8817	5·0088
	0·5	1·4810	2·5741		3·3549		4·2139		4·7912	
	3·5		3·4054	3·7677	4·2389	4·7075	5·0257	5·3861	5·6581	
300	2·5		3·2759	3·6448	4·1110	4·5849	4·9177	5·2957	5·5914	
	1·0		2·9165	3·3058	3·7592	4·2319	4·6183	5·0127	5·3732	5·9630
	0·5		2·6391	3·0447	3·4931	3·9647	4·3891	4·7744	5·1779	
	3·5		3·4446	3·8196	4·2959	4·7744	5·1323	5·5404	5·9162	
600	2·5		3·3133	3·6939	4·1652	4·6458	5·0139	5·4302	5·8157	
	1·0		2·9501	3·3484	3·8073	4·2817	4·6933	5·1087	5·5245	6·5651
	0·5		2·6706	3·0832	3·5373	4·0098	4·4526	4·8524	5·2928	
	3·5		3·4782	3·8625	4·3426	4·8275	5·2112	5·6510	6·1029	
1000	2·5		3·3455	3·7348	4·2100	4·6948	5·0861	5·5273	5·9760	
	1·0		2·9794	3·3844	3·8477	4·3231	4·7517	5·1810	5·6329	7·0088
	0·5		2·6982	3·1161	3·5749	4·0479	4·5032	4·9130	5·3772	

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Table 2

Logarithms (to base 10) of ranges (in centimetres) of particles launched at 66 deg.

Velocity ($m\ s^{-1}$)	Density ($g\ cm^{-3}$)	Radius in centimetres 0·01	0·1	0·3	1·0	3·0	10·0	30·0	100·0	log (vacuum range, cm)
10	3·5	1·7003	2·5946		2·8261		2·8736		2·8786	
	2·5	1·5668	2·5230		2·8073		2·8719		2·8785	
	1·0	1·1906	2·2805		2·7242		2·8601		2·8770	2·8799
	0·5	0·8980	2·0581		2·6222		2·8427		2·8753	
30	3·5	1·9068	2·9670		3·4896		3·8010		3·8290	
	2·5	1·7658	2·8521		3·4235		3·7897		3·8276	
	1·0	1·3775	2·5202		3·1834		3·7329		3·8180	3·8341
	0·5	1·0807	2·2552		2·9752		3·6544		3·8036	
100	3·5	2·0813	3·1260		3·8644		4·5529		4·8358	
	2·5	1·9382	2·9768		3·7468		4·4821		4·8195	
	1·0	1·5462	2·6403		3·4194		4·2548		4·7461	4·8799
	0·5	1·2474	2·3629		3·1673		4·0556		4·6504	
300	3·5		3·2023	3·5759	4·0410	4·5111	4·8673	5·2357	5·5274	
	2·5		3·0702	3·4512	3·9111	4·3852	4·7573	5·1368	5·4571	
	1·0		2·7047	3·1073	3·5575	4·0304	4·4499	4·8381	5·2261	5·8341
	0·5		2·4232	2·8423	3·2906	3·7590	4·2103	4·5920	5·0199	
600	3·5		3·2378	3·6220	4·0926	4·5719	4·9631	5·3820	5·7887	
	2·5		3·1043	3·4949	3·9602	4·4406	4·8427	5·2624	5·6795	
	1·0		2·7359	3·1454	3·6013	4·0758	4·5155	4·9256	5·3690	6·4362
	0·5		2·4528	2·8771	3·3309	3·8005	4·2660	4·6621	5·1253	
1000	3·5		3·2684	3·6605	4·1354	4·6207	5·0348	5·4885	5·9963	
	2·5		3·1339	3·5317	4·0014	4·4857	4·9075	5·3542	5·8474	
	1·0		2·7631	3·1781	3·6384	4·1139	4·5670	4·9921	5·4732	6·8799
	0·5		2·4788	2·9072	3·3654	3·8358	4·3108	4·7173	5·2033	

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6. A note on the 1968 eruption of Arenal

Fudali & Melson (1972) have noted that during the initial explosive phase of the 1968 eruption of the volcano Arenal in Costa Rica, small blocks of density $2{\cdot}6\,\mathrm{g\,cm^{-3}}$ were thrown to distances of up to 5 km, there excavating craters whose size indicated a kinetic energy at impact of 10^{14} erg for the blocks. From a series of approximate calculations, in which the drag coefficient, C, was assumed to be constant for a given size of block, they showed that the launch velocities of such blocks must have been at least $600\,\mathrm{m\,s^{-1}}$ implying a magma chamber pressure of 4700 bars and a total kinetic energy of the initial explosion equal to $2{\cdot}4 \pm 1{\cdot}2 \times 10^{21}$ erg.

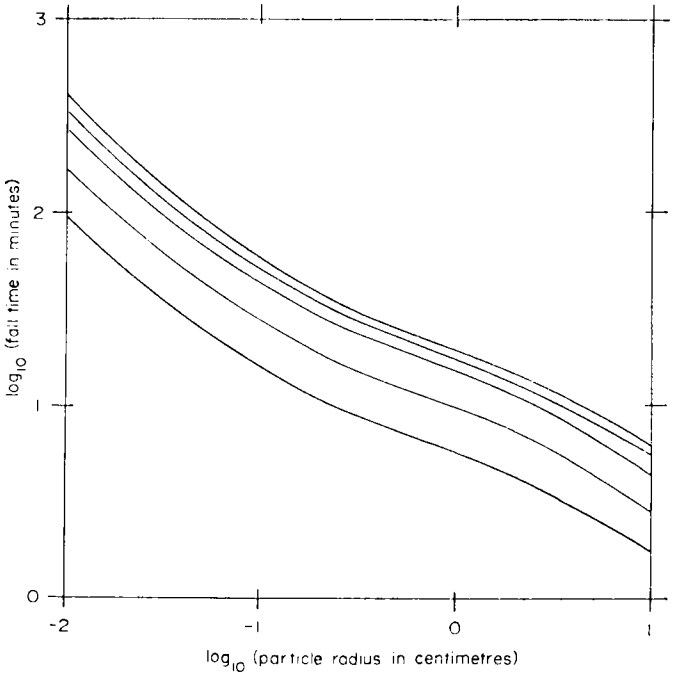

FIG. 2. As Fig. 1, with particle density $1{\cdot}0\,\mathrm{g\,cm^{-3}}$.

The values of C which they assumed are apparently too low. At speeds above $200\,\mathrm{m\,s^{-1}}$, compressibility effects increase the drag coefficient, the maximum factor being about 2 at Mach numbers around 0·95; this effect is neglected in their calculations, as is the variation of C with Reynolds number. Also, in order to obtain minimum values for the launch velocities, they use values of C appropriate to spheres. In the present calculations, the more realistic non-spherical shape deduced in Paper I is used.

Using Table 1, interpolating for particle density equal to $2{\cdot}6\,\mathrm{g\,cm^{-3}}$, the values in Table 4 are found. Final kinetic energies were obtained from the computer program described in Section 4. A kinetic energy at impact of 10^{14} erg, at a range of 5 km, corresponds to the launching of an 88·6-cm diameter block at $1090\,\mathrm{m\,s^{-1}}$. This velocity is almost double that found by Fudali & Melson, and, if used to calculate the magma chamber pressure and total kinetic energy of the explosion would lead to values greater than theirs by a factor of 3·35 in each case. Yet by comparing the Arenal explosion crater with the Sedan nuclear event, Fudali & Melson show that the value of total kinetic energy they derive is apparently correct.

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Table 3

Logarithms (to base 10) of ranges (in centimetres) of particles launched at 87 deg.

Velocity ($m\ s^{-1}$)	Density ($g\ cm^{-3}$)	Radius in centimetres 0·01	0·1	0·3	1·0	3·0	10·0	30·0	100·0	log (vacuum range, cm)
10	3·5	0·8395	1·7530		1·9790		2·0211		2·0268	
	2·5	0·7028	1·6816		1·9614		2·0207		2·0267	
	1·0	0·3195	1·4373		1·8830		2·0099		2·0254	2·0279
	0·5	0·0220	1·2204		1·7846		1·9937		2·0236	
30	3·5	1·0359	2·1219		2·6641		2·9505		2·9775	
	2·5	0·8312	2·0048		2·5897		2·9395		2·9762	
	1·0	0·4997	1·6670		2·3479		2·8843		2·9673	2·9822
	0·5	0·2001	1·3979		2·1373		2·8070		2·9540	
100	3·5	1·2041	2·2716		3·0228		3·7156		3·9873	
	2·5	1·0596	2·1415		2·9034		3·6437		3·9721	
	1·0	0·6644	1·7793		2·5717		3·4116		3·9026	4·0279
	0·5	0·3640	1·4991		2·4068		3·2078		3·8097	
300	3·5		2·3424	2·7211	3·1877	3·6669	4·0293	4·3988	4·6906	
	2·5		2·2090	2·5955	3·0568	3·5394	3·9157	4·3000	4·6212	
	1·0		1·8396	2·2492	2·6997	3·1770	3·5993	4·0006	4·3900	4·9822
	0·5		1·5554	1·9823	2·4302	2·9015	3·3549	3·7521	4·1815	
600	3·5		2·3756	2·7638	3·2356	3·7238	4·1204	4·5432	4·9575	
	2·5		2·2409	2·6361	3·1024	3·5912	3·9964	4·4227	4·8464	
	1·0		1·8690	2·2847	2·7404	3·2191	3·6607	4·0836	4·5308	5·5843
	0·5		1·5834	2·0148	2·4679	2·9400	3·4069	3·8174	4·2831	
1000	3·5		2·4043	2·7997	3·2755	3·7693	4·1881	4·6482	5·1767	
	2·5		2·2687	2·6703	3·1407	3·6332	4·0573	4·5119	5·0191	
	1·0		1·8948	2·3152	2·7752	3·2547	3·7089	4·1463	4·6333	6·0279
	0·5		1·6082	2·0430	2·5003	2·9732	3·4489	3·8687	4·3578	

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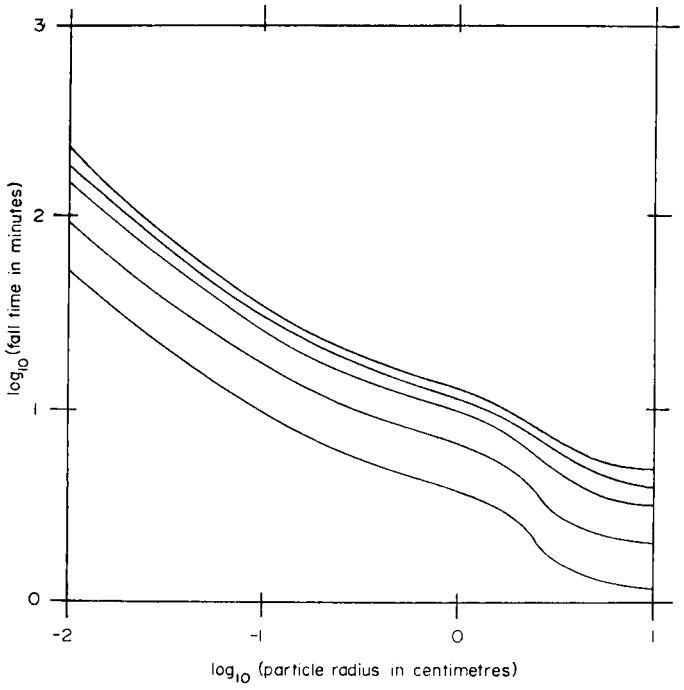

FIG. 3. As Fig. 1, with particle density 2·5 g cm^{-3}.

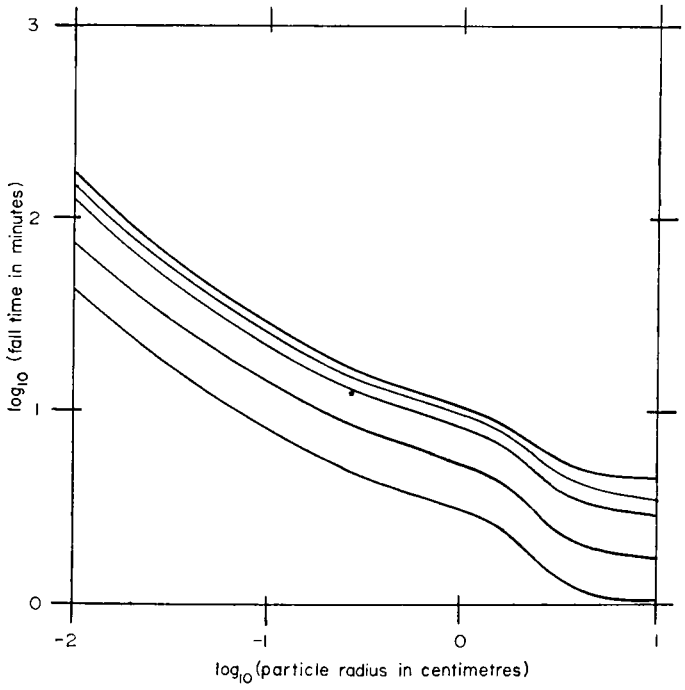

FIG. 4. As Fig. 1, with particle density 3·5 g cm^{-3}.

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Table 4

Parameters of blocks with density 2·6 g cm^{-3} reaching 5 km when launched at elevation 45°

Diameter (m)	Launch velocity (m s^{-1})	Final kinetic energy (erg)
1·60	488	$10^{14.89}$
1·24	660	$10^{14.50}$
1·00	870	$10^{14.19}$
0·60	2760	$10^{13.39}$

The solution to this dilemma seems to lie in consulting their Fig. 5. This figure gives the energy required to excavate a secondary crater as a function of the displaced mass of crater material. Fudali & Melson estimate the displaced mass from a typical secondary crater to be $1{\cdot}2 \times 10^6$ g, and deduce that this corresponds to a cratering energy of about 10^{14} erg. In fact, careful examination of the figure shows that a value of $10^{14.5}$ would be more accurate. Table 4 (of this paper) demonstrates that the launch velocity needed for blocks to reach 5 km varies rapidly with final kinetic energy. Using $10^{14.5}$ instead of 10^{14} erg for the final kinetic energy of blocks, the best solution corresponding to the launching of a 1·24-m diameter block at 660 m s^{-1}. This velocity is, as it happens, very close to that found by Fudali & Melson (600 m s^{-1}), so that the present calculations may be regarded as confirming their estimates of magma chamber pressure and total kinetic energy.

7. Discussion of limits of applicability of computed quantities

Some care is needed in the use of these ranges and fall times. The fall times given only apply if there are no vertical winds. Fig. 1 shows that, for particles of density 0·5 g cm^{-3} (the most sensitive to wind action) the average speeds while falling from heights of 5 and 50 km are about 42 m s^{-1} and 100 m s^{-1} respectively for a 10 cm radius particle and about 0·5 and 1·0 m s^{-1} for a 0·01-cm radius particle. In the lower atmosphere (up to 15 km), at least, horizontal winds normally have speeds of order 1 to 15 m s^{-1}; vertical winds have speeds at least an order of magnitude less than these (Berry, Bollay & Bears 1945), say 0·1–1 m s^{-1}. Clearly, the fall time of a 10-cm radius particle will be only slightly affected by winds. The time of fall of a 0·01 radius particle may, however, be altered substantially. The effect will be minimized if the particle encounters both upward and downward air currents during its fall. This factor should be borne in mind when applying the fall times to actual problems.

As mentioned earlier, the ranges of large particles will be little affected by the stream of erupted gas near the vent since (a) the extra drag force on such particles is relatively small compared with gravity, and (b) it acts over only a small fraction of the total trajectory. However, the ranges given for all particles neglect the peculiar conditions near the vent. The values given for small particles should, therefore, only be regarded as estimates of the distances they travel after leaving the influence of the eruption column. If they are carried upwards for any appreciable distance by the eruption column, the values given will, in any case, be irrelevant, since it is no longer appropriate to think of the small particles as being launched from the vent with a specific velocity and elevation.

Subject to these limitations, the ranges and fall times presented here will be applied in later papers to the analysis of specific eruptions, using the methods outlined in Section 2.

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